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for Hadron and Lepton Colliders
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Two-loop electroweak logarithms at high energies

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Based on works by

- **B. Feucht/Jantzen, J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov**
  e.g. Nucl. Phys. B 731 (2005) 188
- **A. Denner, B. Jantzen, S. Pozzorini** hep-ph/0608326 (→ Nucl. Phys. B)

Overview

I Electroweak corrections at high energies

II Four-fermion scattering @ $\mathrm{N^3LL}$ via evolution equations

- factorization of the 4-fermion amplitude
- form factor contributions

III Arbitrary high-energy processes @ NLL

- extraction of mass-singular logs at 1 and 2 loops
- factorizable and non-factorizable contributions
- result for massless fermionic processes

IV Summary & comparison

I Electroweak (EW) corrections at high energies

EW collider experiments

- today (LEP, Tevatron): relevant energies $\lesssim M_{\mathrm{W,Z}}$
- upcoming colliders (LHC, ILC) $\longrightarrow$ explore TeV regime

  $\hookrightarrow$ new energy domain $\sqrt{s} \gg M_{\mathrm{W}}$ becomes accessible!

EW radiative corrections at high energies $\sqrt{s} \gg M_{\mathrm{W}}$

$\Rightarrow$ enhanced by large Sudakov logarithms

$$\ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \sim 25 \quad \text{at } \sqrt{s} \sim 1\,\mathrm{TeV}$$

Logs present in exclusive observables with only virtual W and Z bosons (this talk), but also in inclusive observables due to Bloch–Nordsieck violations

Ciafaloni, Ciafaloni, Comelli '00,'01; Ciafaloni, Comelli '05

General form of EW corrections for $s \gg M_\mathrm{W}^2$

Perturbative expansion in powers of $\frac{\alpha}{4\pi\sin^2\theta_\mathrm{w}} \approx 0.003$

Asymptotic expansion in powers of $\frac{M_\mathrm{W}^2}{s}$ **and** powers of $L = \ln\left(\frac{s}{M_\mathrm{W}^2}\right)$

1 loop:
$$\frac{\alpha}{4\pi}\Big[\underset{\substack{\downarrow\\ -17\,\%}}{C_1^{\mathrm{LL}}\,L^2} + \underset{\substack{\downarrow\\ +12\,\%}}{C_1^{\mathrm{NLL}}\,L} + \underset{\substack{\downarrow\\ -3\,\%}}{C_1^{\mathrm{N^2LL}}}\Big] + \mathcal{O}\left(\frac{M_\mathrm{W}^2}{s}\right)$$

$\sigma(u\bar{u} \to d\bar{d})$ @ $\sqrt{s} = 1$ TeV

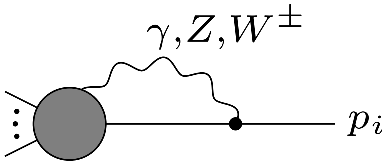
2 loops:
$$\left(\frac{\alpha}{4\pi}\right)^2\Big[\underset{\substack{\downarrow\\ +1.7\,\%}}{C_2^{\mathrm{LL}}\,L^4} + \underset{\substack{\downarrow\\ -1.8\,\%}}{C_2^{\mathrm{NLL}}\,L^3} + \underset{\substack{\downarrow\\ +1.2\,\%}}{C_2^{\mathrm{N^2LL}}\,L^2} + \underset{\substack{\downarrow\\ -0.3\,\%}}{C_2^{\mathrm{N^3LL}}\,L} + C_2^{\mathrm{N^4LL}}\Big] + \mathcal{O}\left(\frac{M_\mathrm{W}^2}{s}\right)$$

Theoretical prediction with accuracy $\sim 1\,\%$ required

$\Rightarrow$ 2-loop corrections important

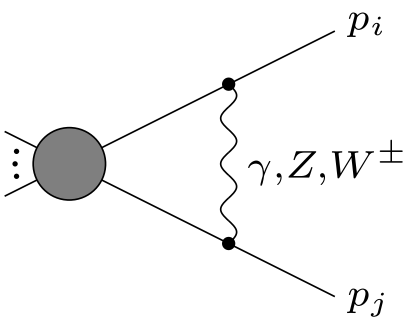
$\Rightarrow$ 2-loop LL approximation not sufficient, need also subleading logs

Origin of logarithms $\ln(s/M_W^2)$ in virtual corrections



mass singularities from virtual gauge bosons $(\gamma, Z, W^\pm)$ coupling to on-shell external leg

$\longrightarrow$ single logs from collinear region



special case:
gauge bosons exchanged between **2** on-shell external legs
$\longrightarrow$ double logs from soft–collinear region

- for massless photons: $\log \rightsquigarrow \dfrac{1}{\epsilon}$ in $D = 4 - 2\epsilon$ dimensions

  $\Rightarrow$ count $1/\epsilon$ poles like logs for logarithmic approximations (LL, NLL, ...)

- additional single logs and poles from ultraviolet singularities

- EW 1-loop LLs & NLLs for arbitrary processes are universal

  $\hookrightarrow$ depend only on quantum numbers of external particles — Denner, Pozzorini '00, '01

Approaches for virtual two-loop EW corrections at high energies

Resummation of 1-loop result to all orders:

- LL for arbitrary processes — Fadin, Lipatov, Martin, Melles '99
- NLL for arbitrary processes ($M_Z = M_W$) — Melles '00, '01
- **N²LL for massless $f\bar{f} \to f'\bar{f}'$** ($M_Z = M_W$) — Kühn, Penin, Smirnov '99, '00; Kühn, Moch, Penin, Smirnov '01

$\longrightarrow$ apply evolution equations to spontaneously broken SU(2)×U(1) EW model

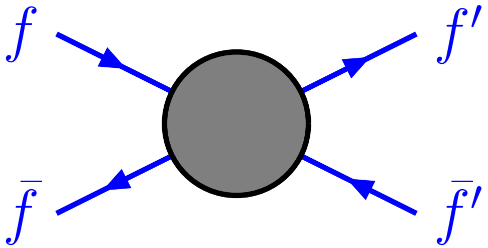
$\hookrightarrow$ rely on splitting of EW theory into symmetric SU(2)×U(1) and QED regime

Diagrammatic 2-loop calculations to check & extend resummation predictions:

- LL for fermionic form factor — Melles '00; Hori, Kawamura, Kodaira '00
- LL for arbitrary processes — Beenakker, Werthenbach '00, '01
- angular-dependent NLLs for arbitrary processes — Denner, Melles, Pozzorini '03
- complete NLL for fermionic form factor — Pozzorini '04
- **N³LL for massless fermionic form factor** ($M_Z = M_W$)
  $\hookrightarrow$ **N³LL for massless $f\bar{f} \to f'\bar{f}'$** ($M_Z \approx M_W$) via evolution equations — B.J., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04, '05
- **NLL for arbitrary massless fermionic processes** ($M_Z \neq M_W$) — Denner, B.J., Pozzorini '06

II Four-fermion scattering @ N^{3}LL

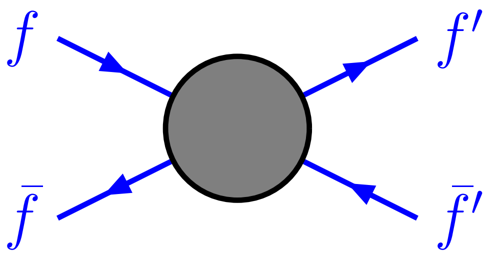
Factorization of QED contributions



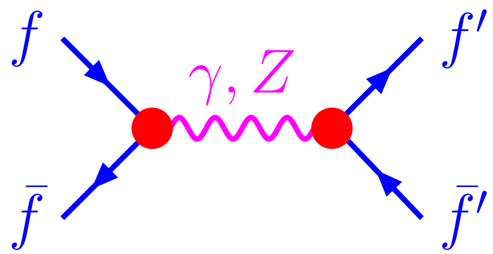
$$A = U_{\text{QED}} \cdot A_{\text{EW}}$$

- QED factor $U_{\text{QED}} \rightarrow$ soft/collinear singularities from virtual massless photons
- $A_{\text{EW}} \rightarrow$ remaining electroweak contributions, safe from photonic singularities
- calculate A_{EW} by evaluating A/U_{QED} with $M_\gamma = M_{\text{W}}$
  ↪ works at N^{3}LL accuracy if SU(2)↔U(1) mixing is neglected
  B.J., Kühn, Penin, Smirnov '04, '05
- single mass parameter: $M_{\text{Z}} = M_\gamma = M_{\text{W}}$
- include mass difference $(M_{\text{Z}} - M_{\text{W}})$ by expansion around $M_{\text{Z}} \approx M_{\text{W}}$

Factorization into form factor and reduced amplitude

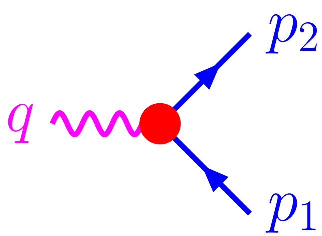


$$A_{\mathsf{EW}} = \frac{g^2}{s} F^2 \tilde{A}$$



$$\times \tilde{A}$$

Form factor F of vector current:



$$= F \cdot \bar{u}(p_2)\, \gamma^\mu\, u(p_1) + \mathcal{O}(\text{fermion masses})$$

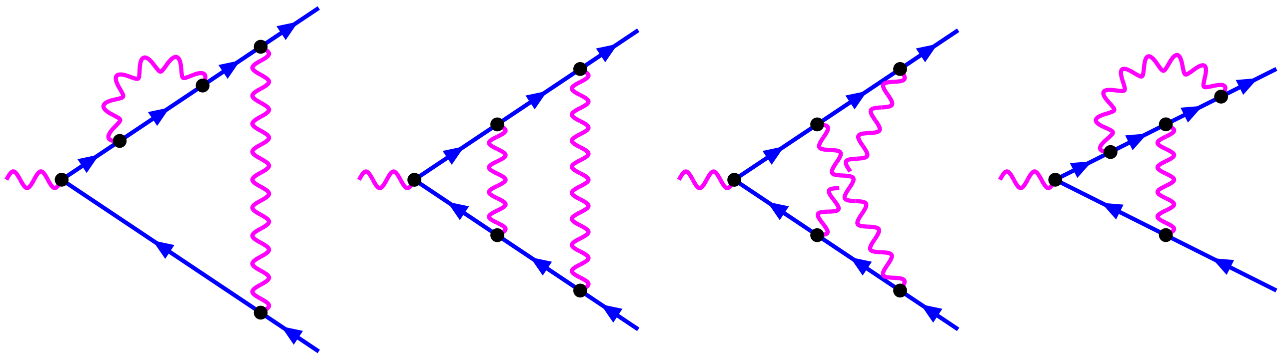
High-energy behaviour $s \sim |t| \sim |u| \gg M_{\mathrm{W}}^2$ references: see Kühn et al. '01

- all double logs $\alpha^n \ln^{2n}$ $\rightsquigarrow$ form factors F^2
- reduced amplitude $\tilde{A}$ $\longrightarrow$ only single logs $\alpha^n \ln^n$
- evolution equations $\longrightarrow$ $\partial F/\partial s$, $\partial \tilde{A}/\partial s$

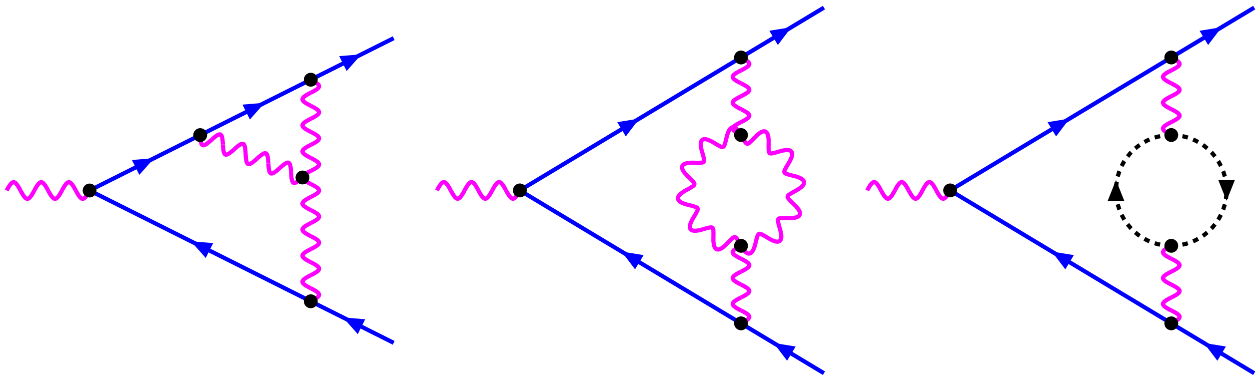
For full logarithmic (N^{3}LL) 2-loop amplitude
$\hookrightarrow$ need 2-loop vertex contributions to form factor F

Vertex diagrams for the SU(2) form factor at two loops

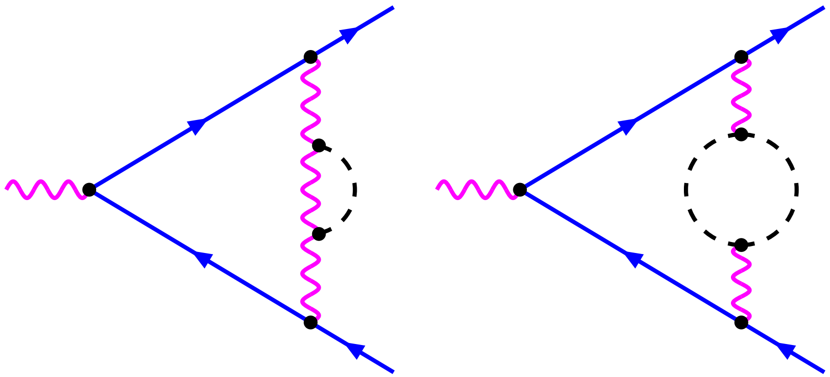
Abelian:
$\hookrightarrow$ U(1)



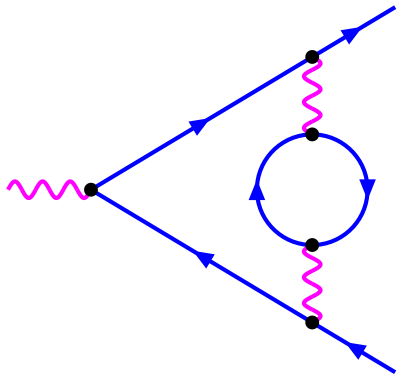
non-Abelian:



Higgs:



fermion:

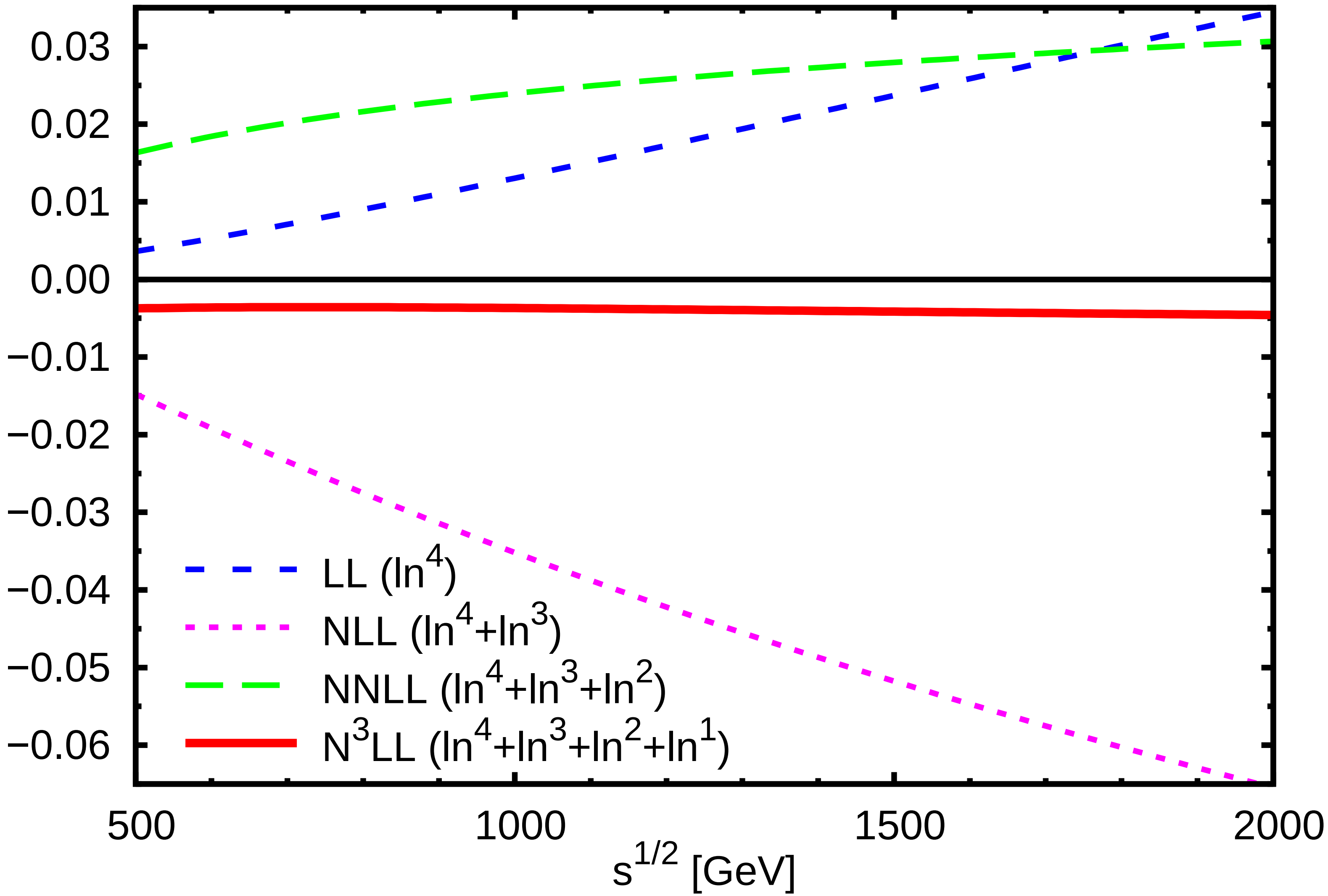


+ 1-loop×1-loop corrections + renormalization

Example for EW results: cross section $\sigma(e^+e^- \to q\bar{q})$ $(q = d, s)$

numerical 2-loop result:

$$\left(\frac{\alpha}{4\pi s_{\mathrm{w}}^2}\right)^2 \left[+2.79\, L^4 - 51.98\, L^3 + 321.34\, L^2 - 757.35\, L \right]$$



III Arbitrary high-energy processes @ NLL

Goal: virtual 2-loop EW corrections for arbitrary processes in NLL accuracy

↪ diagrammatic approach: not rely on evolution equations, but check them

↪ provide process-independent corrections for arbitrary $2 \to n$ reactions

Implement

- different large kinematical invariants $|(p_i + p_j)^2| \sim Q^2 \gg M_\mathrm{W}^2$
- different heavy particle masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{top}^2 \sim M_\mathrm{Higgs}^2$ (light masses = 0)

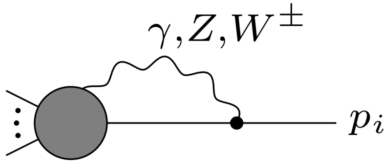
⇒ Logs $L = \ln\left(\frac{Q^2}{M_\mathrm{W}^2}\right)$ and $\frac{1}{\epsilon}$ poles (from virtual photons)

1 loop: LL $\to \epsilon^{-2},\ L\epsilon^{-1},\ L^2,\ L^3\epsilon,\ L^4\epsilon^2$; NLL $\to \epsilon^{-1},\ L,\ L^2\epsilon,\ L^3\epsilon^2$

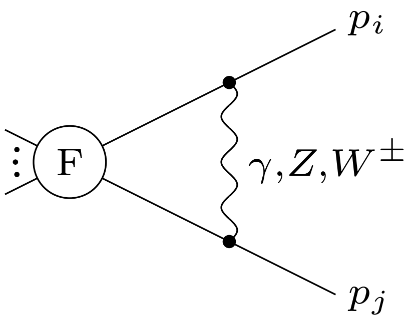
2 loops: LL $\to \epsilon^{-4},\ L\epsilon^{-3},\ L^2\epsilon^{-2},\ L^3\epsilon^{-1},\ L^4$; NLL $\to \epsilon^{-3},\ L\epsilon^{-2},\ L^2\epsilon^{-1},\ L^3$

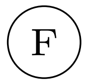
⇒ NLL coefficients involve small logs $\ln\left(\frac{|(p_i+p_j)^2|}{Q^2}\right)$ and $\ln\left(\frac{M_\mathrm{Z}^2, m_\mathrm{top}^2, M_\mathrm{Higgs}^2}{M_\mathrm{W}^2}\right)$

Extraction of NLL mass singularities at one loop

Contributions originate from 

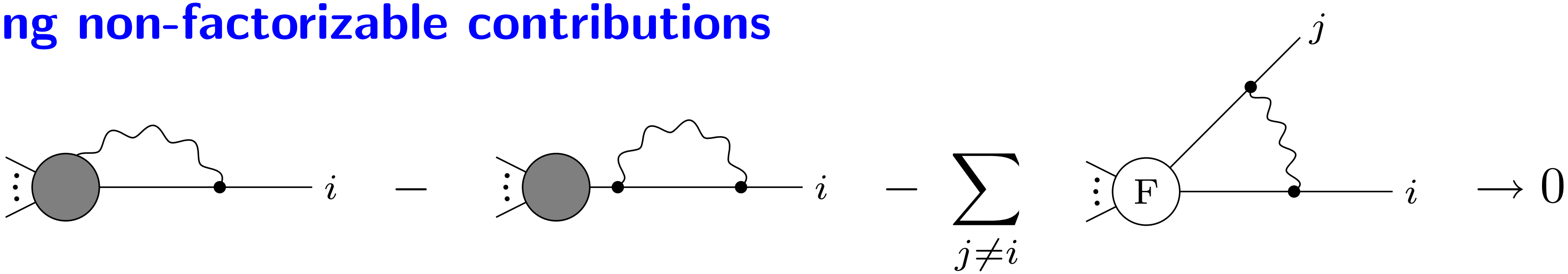
in the collinear region

Isolate factorizable contributions: 

- gauge boson momentum set to zero in tree subdiagram 

- soft–collinear approximation eliminates Dirac structure of loop corrections

Remaining non-factorizable contributions



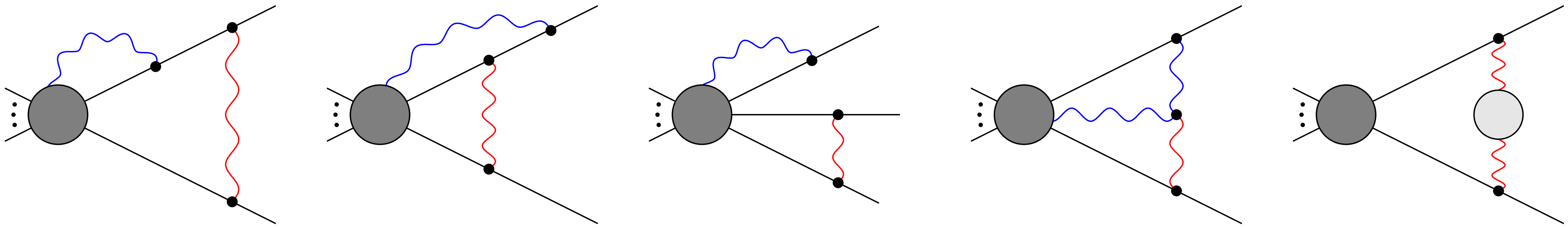
vanish due to collinear Ward identities

Denner, Pozzorini '00, '01

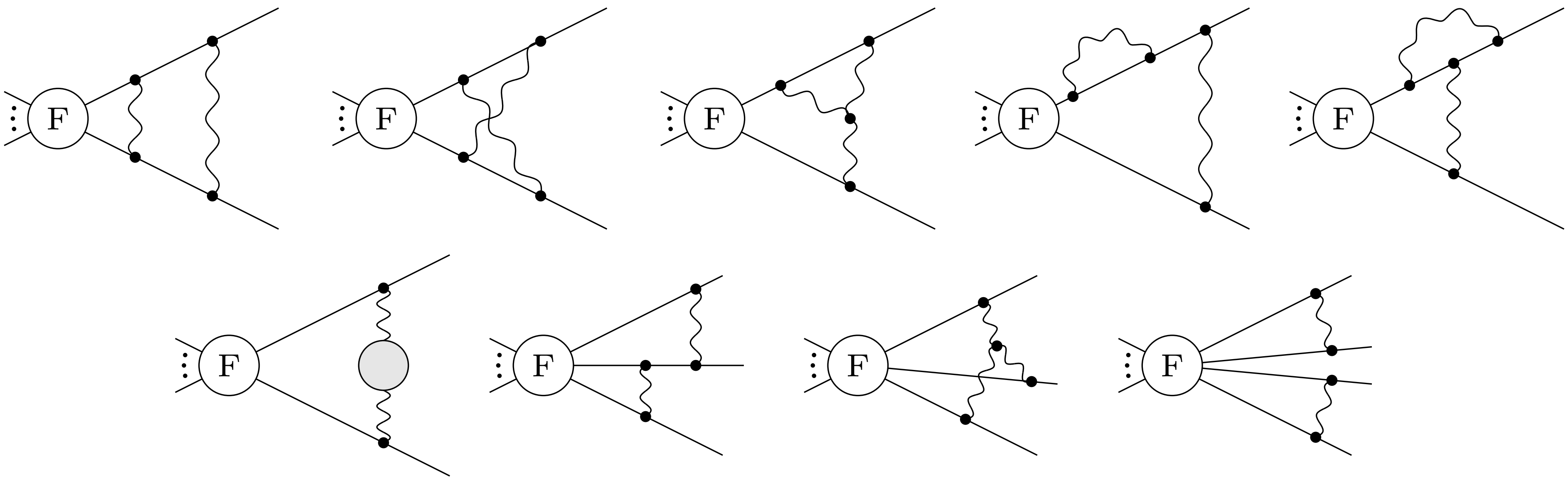
The factorizable contributions contain all soft and/or collinear NLL mass singularities.

Extraction of NLL mass singularities at two loops

$\hookrightarrow$ soft×soft and soft×collinear contributions:



Factorizable contributions (including soft×UV contributions):



- calculated with soft–collinear approximation and projection techniques
- remaining non-factorizable contributions vanish due to collinear Ward identities

Denner, B.J., Pozzorini '06

NLL result for massless fermionic processes $f_1 f_2 \to f_3 \cdots f_n$ up to two loops

Factorizable contributions calculated & checked with 2 independent methods:

- automatized algorithm based on sector decomposition — Denner, Pozzorini '04
- combination of expansion by regions & Mellin–Barnes representations — Beneke, Smirnov '98; Ussyukina '75; Boos, Davydychev '91; B.J., Smirnov '06

Universal correction factors: $$\boxed{\mathcal{M} = \mathcal{M}_0\, F^{\text{sew}}\, F^{\text{Z}}\, F^{\text{em}}}$$

symmetric-electroweak factor: $$F^{\text{sew}} = \exp\left[\frac{\alpha}{4\pi}\, F_1^{\text{sew}} + \left(\frac{\alpha}{4\pi}\right)^2 G_2^{\text{sew}}\right]$$

electromagnetic factor: $$F^{\text{em}} = \exp\left[\frac{\alpha}{4\pi}\, \Delta F_1^{\text{em}} + \left(\frac{\alpha}{4\pi}\right)^2 \Delta G_2^{\text{em}}\right]$$

terms from $M_\text{Z} \neq M_\text{W}$: $$F^{\text{Z}} = 1 + \frac{\alpha}{4\pi}\, \Delta F_1^{\text{Z}}$$

- F^{sew} equals result from symmetric SU(2)×U(1) theory with $M_\gamma = M_\text{W} = M_\text{Z}$
- electromagnetic terms in F^{em} factorize and exponentiate separately
  $\hookrightarrow$ separation of photonic singularities possible

Exponentiated one-loop terms: LLs & NLLs

$$F_1^{\text{sew}} = -\frac{1}{2}\left(L^2 + \frac{2}{3}L^3\epsilon + \frac{1}{4}L^4\epsilon^2 - 3L - \frac{3}{2}L^2\epsilon - \frac{1}{2}L^3\epsilon^2\right)\sum_{i=1}^{n}\left(\frac{Y_i^2}{4c_{\text{w}}^2} + \frac{C_i}{s_{\text{w}}^2}\right)$$
$$+\left(L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\ln\left(\frac{-(p_i+p_j)^2}{Q^2}\right)\sum_{V=\gamma,Z,W^\pm} I_i^{\bar{V}} I_j^V$$

$$\Delta F_1^{\text{em}} = -\frac{1}{2}\left(2\epsilon^{-2} - L^2 - \frac{2}{3}L^3\epsilon - \frac{1}{4}L^4\epsilon^2 + 3\epsilon^{-1} + 3L + \frac{3}{2}L^2\epsilon + \frac{1}{2}L^3\epsilon^2\right)\sum_{i=1}^{n} Q_i^2$$
$$-\left(\epsilon^{-1} + L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\ln\left(\frac{-(p_i+p_j)^2}{Q^2}\right) Q_i Q_j$$

$$\Delta F_1^{\text{Z}} = \left(L + L^2\epsilon + \frac{1}{2}L^3\epsilon^2\right)\ln\left(\frac{M_{\text{Z}}^2}{M_{\text{W}}^2}\right)\sum_{i=1}^{n}\left(\frac{c_{\text{w}}}{s_{\text{w}}}T_i^3 - \frac{s_{\text{w}}}{c_{\text{w}}}\frac{Y_i}{2}\right)^2$$

Additional two-loop terms with one-loop β-function coefficients: only NLLs

$$G_2^{\text{sew}} = \frac{1}{6}L^3\sum_{i=1}^{n}\left(b_1^{(1)}\frac{Y_i^2}{4c_{\text{w}}^2} + b_2^{(1)}\frac{C_i}{s_{\text{w}}^2}\right)$$

$$\Delta G_2^{\text{em}} = \left(\frac{3}{4}\epsilon^{-3} + L\epsilon^{-2} + \frac{1}{2}L^2\epsilon^{-1}\right) b_{\text{QED}}^{(1)}\sum_{i=1}^{n} Q_i^2$$

$[\mu_{\text{R}}^2 = M_{\text{W}}^2]$

IV Summary & comparison

Four-fermion scattering $f\bar{f} \to f'\bar{f}'$

B.J., Kühn, Moch, Penin, Smirnov

- factorization, evolution equations
- form factor corrections confirm & extend resummation results
- N^{3}LL EW 2-loop corrections ($M_\mathrm{Z} \approx M_\mathrm{W}$, SU(2)↔U(1) mixing neglected)

Massless fermionic processes $f_1 f_2 \to f_3 \cdots f_n$

Denner, B.J., Pozzorini

with different $|(p_i + p_j)^2| \gg M_\mathrm{W}^2$ and different masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{top}^2 \sim M_\mathrm{Higgs}^2$:

- extraction of mass singularities via factorizable contributions
- NLL EW 2-loop corrections in $D = 4 - 2\epsilon$ dimensions
- agreement with resummation results
- method generalizable to arbitrary processes

Talking about asymptotic expansions . . .

“Historic” example: light cone expansion

In dieser Arbeit sollen zwei Arten der Lichtkegelentwicklung im Rahmen einer Feldtheorie vom Wightman-Typ untersucht werden.

Zu diesem Zweck wird zunächst eine Definition einer Lichtkegel-Entwicklung angegeben - in Analogie zur Entwicklung von Distributionen an singulären Punkten. Wir nehmen an, daß für das Produkt $A(X-x/2)\,A(X+x/2)$ eine Entwicklung der folgenden Form möglich ist:

$$(1)\quad \int A(X-x/2)\,A(X+x/2)\,f(X)\,g(x)\,\tfrac{1}{\varrho}\,h(x^2/\varrho)\,dX\,dx = \sum_{i=1}^{n} s_i(\varrho)\,B_i(f,g,h) + R_\varrho(f,g,h),$$

wobei die s_i nach der Stärke ihrer Singularität für $\varrho \to 0$ geordnet sind und $R_\varrho / s_n(\varrho)$ gegen Null geht.

Source:

Operatorprodukt-Entwicklungen am Lichtkegel

Inaugural-Dissertation

zur Erlangung des Doktorgrades

der Fakultät für Physik

der Ludwig-Maximilians-Universität

München

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